

Today I'm going to talk about a proposed experiment about Kepler's law of harmonies that you can do with your students.

My name is Takuo Takarada. Please call me TT next time we meet on the road. I teach physics at a high school in Osaka, the second most populous area in Japan.

By the way have you ever intuited the ratio of planet size, mass, revolution radius, or gravity? That's not really something you encounter in everyday life. I teach my students about these things in class, but they used to find it very hard to understand. So I came up with an experiment that lets you feel the ratio of planet size, mass, revolution radius, and universal gravitation. This experiment is an applying a uniform circular motion experiment.

Tycho Brahe 1546-1601
Denmark

Johannes Kepler 1571-1630
Germany

1600 - Momentous joint research in Prague

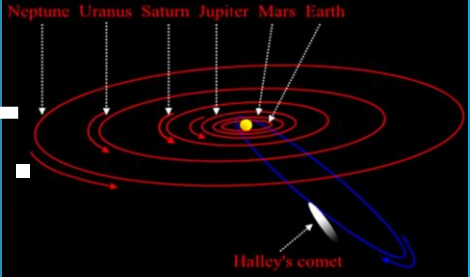
**1609 - Kepler discovers the law of harmonies
Scientifically prove the heliocentric theory**

The left picture is Tycho Brahe. He was an astronomer who accurately and repeatedly measured the planet's positions for 40 years. The right picture is Johannes Kepler. He was a mathematically talented astronomer who tried to figure out the mechanism behind beautiful outer space. As fate would have it, they met in 1600. Tycho passed away one year later, but about 20 years down the road, Kepler derived the law of harmonies and scientifically proved the theory of heliocentrism, which today we take for granted.

T^2 ← The square of the period

$\frac{T^2}{a^3} = \text{CONST.}$

← the cube of the long radius

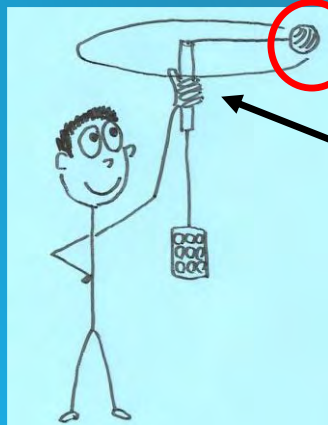


• The square of the period T is proportional to the cube of the long radius a .

The diagram shows a central yellow sun with several red elliptical orbits. Labels at the top point to the orbits: Neptune, Uranus, Saturn, Jupiter, Mars, and Earth. A blue comet, labeled 'Halley's comet', is shown in a highly elliptical orbit that crosses the inner solar system.

Kepler's law of harmonies goes like this. The square of the period of a star orbiting around the sun and the cube of the long radius are proportional to each other. Because Earth, Jupiter, and Halley's Comet orbit the sun, he found that they have the same value.

How to Experiment with the Great, Majestic Kepler's Law in the Classroom!



← This ball represents a planet.

← Spin it over your head.

This is how you can experiment with Kepler's great, majestic law in the classroom. This ball represents a planet. Spin it over your head. That's all. And it's so simple.



This is the experimental equipment we use. It's a planet made of resin clay. From right to left, we have Mars, Earth, Venus, Mercury, and finally the hypothetical Planet X. Next, we have a weight. It corresponds with the universal gravitation that planets receive from the sun. We also have a glass tube. Both ends are heat-treated for safety. If you put a string through it, you can make the planet move in a circular motion. We also have some fishing lines. It is used to connect the planet and the weight. And we have one last thing: a stopwatch. It is used to measure the orbital period of a planet.

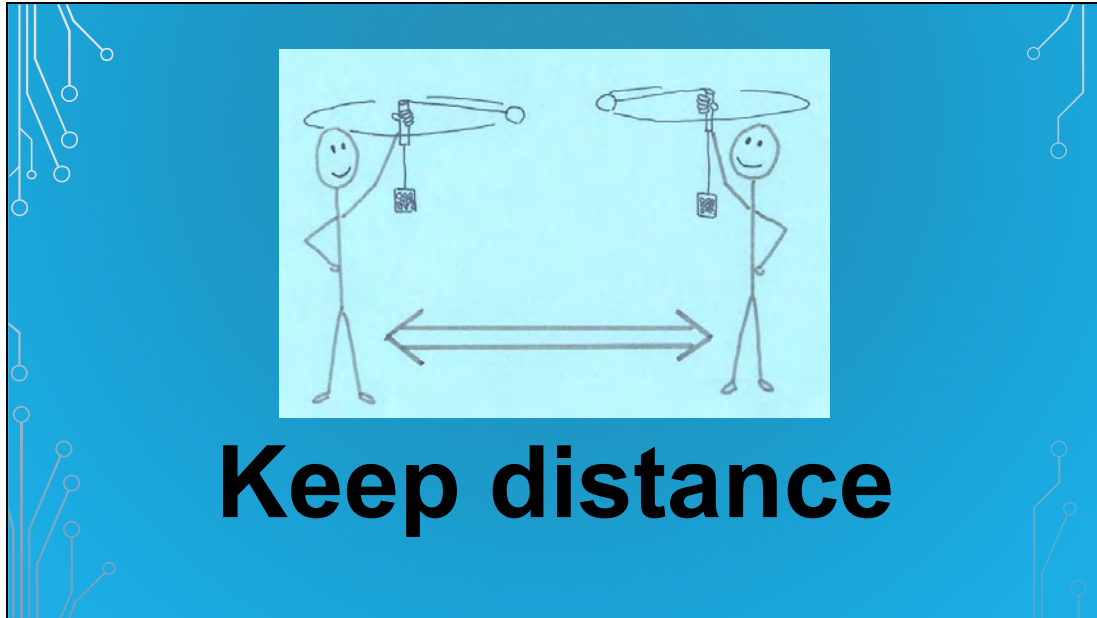
I will show you how to make a planet. Boil water in a large pot and add the clay that has been weighted and shaped. Boil for about 30 minutes to harden.

Planetary Fact Sheet

gw, gram-weight

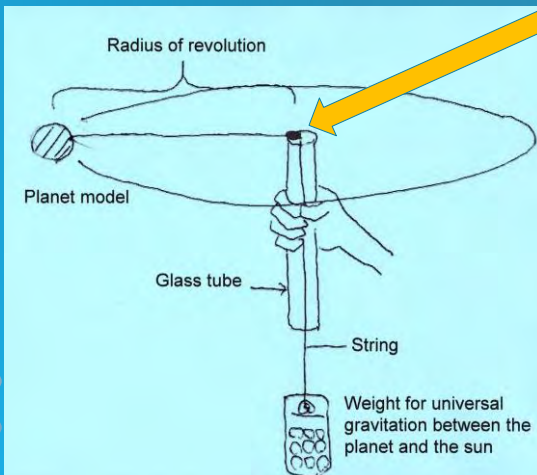
	mass(g)	Revolution radius(m)	Universal gravitation(gw)
Mercury			
Venus			
PlanetX			
Earth	100	0.45	270
Mars			

The mass of the weight corresponding to universal gravitation used in this experiment was calculated using data from the Planetary Fact Sheet. The basis was 100 g for the Earth's mass, 0.45 m for the orbital radius, and 270 gw for the universal gravitation. I wanted to try Jupiter, too, but when I estimated its values using this ratio, they were too big: 86 kg for the mass and 2.3 m for the radius of revolution. So I gave up.



The most important thing I want you to be careful of in today's experiment is to keep your distance so that your planet doesn't hit the person next to you. Now let's begin. If you are worried, wear a helmet and goggles.

EXPERIMENTAL METHOD DIAGRAM



Spin it at a speed where the black mark remains stable at the upper end of the glass tube.

Keep your body and hands away from the string and the weight.

Ready, go! Spin it once, twice, three times.... and measure the time it takes for the model to spin around ten times.

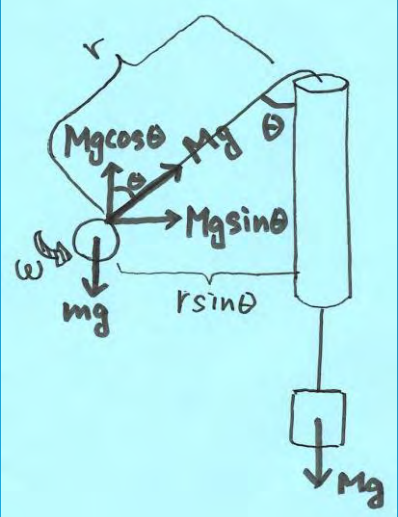
Let me explain the experiment. The three members of a group will work together to carry out the experiment. On the fishing line, there is a black mark made with a marker just at the position of the revolution radius. Let's practice for a while until the rotation settles over your head. If it's too fast, the part with the black mark will come out of the glass tube. If the rotation is too slow, it will be hidden in the glass tube. Don't touch the string or weight. When the rotation settles, measure the time it takes for 10 rotations.



My students will show you an example. One of them will spin it over her head. The other will measure the time it takes for 10 rotations.

Well, I was planning to have everyone enjoy the experiment, but today I will do the experiment for you.

Mercury rotates very fast. When the speed slows down, Mercury falls to the sun like this. Mars is slow to rotate and has a conical motion in this experiment.



The diagram illustrates a conical pendulum. A mass m is attached to a string of length r that makes an angle θ with the vertical. The forces acting on the mass are tension Mg along the string and gravity mg vertically downwards. The tension is decomposed into a vertical component $Mg \cos \theta$ and a horizontal component $Mg \sin \theta$. The horizontal component provides the centripetal force for the circular motion of radius $r \sin \theta$. The angular velocity is denoted by ω . A separate mass M is shown hanging vertically from a support.

$mr\omega^2 \sin \theta = Mg \sin \theta$

$\therefore \omega = \sqrt{Mg / mr}$

The angle θ does not affect the period.

Does conical movement affect the period? Conical motion does not affect the rotation period. This is the motion equation written for the circular motion of the planet. Solving this gives us an angular velocity independent of θ . In this experiment, the angular velocity does not change with a magnitude of θ . However, when θ becomes small, the tension is small compared to the frictional force, so the radius is hard to change. Experiment many times and take the average, like Tycho Brahe. Do you remember. Tycho Brahe made these measurements many times over 40 years and obtained accurate values.

PROCESSING THE RESULTS

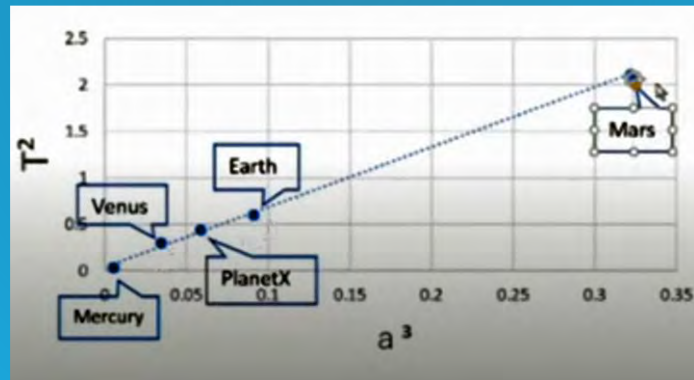
- Enter the measurement results of the square of the period and the cube of the revolution radius into Excel.
- Convert the data into a scatter chart graph.
- Draw an approximation line and make sure that the data for each planet falls on the straight line.

Now we are going to process the data. Then, we'll investigate the relationship between the square of the period and the cube of the revolution radius from the data gathered by each group. First, enter the data into Excel. Next, convert it into a scatter chart graph.

Experimental result													
	a	Radius(m)	T	Period(s)	a ³	T ²							
Mercury	0.174				0.005268024	0							
Venus	0.325				0.034328125	0							
Planet-X	0.388				0.058411072	0							
Earth	0.45				0.091125	0							
Mars	0.685				0.321419125	0							
Average of the period													
	A	B	C	D	E	F	G	H	I	J	AVERAGE		
Mercury												#DIV/0!	
Venus												#DIV/0!	
Planet-X												#DIV/0!	
Earth												#DIV/0!	
Mars												#DIV/0!	

Now, let's enter the data of period. Next let's make this into a scatter chart.

Graph of the relationship between T^2 and a^3



The vertical axis shows the square of the period. And horizontal axis shows the cube of the radius. You can see that it is proportional. This is Mercury Venus Planet X The earth and Mars.

NOW YOU'RE JUST LIKE KEPLER!!



This experiment verifies the law of harmonies based on the law of universal gravitation.

- You can sense the grandeur of Kepler's achievements.**
- You can sense that the law of universal gravitation is accurate.**

Now you're just like Kepler. Kepler discovered his third law without using the law of universal gravitation.

ALSO!!

You can feel a terrestrial planet.

- 1. To be specific, you can feel the ratio of the planet's size and mass.**
- 2. You can feel the ratio of the radius of revolution.**
- 3. You can feel the ratio of universal gravitation produced by the sun.**
- 4. You can feel the ratio of the orbital period.**

Also, when you get the planet models, you will be able to feel the ratio of the size, mass, revolution radius, and universal gravitation of each planet.

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I hope you'll have your students experience Kepler's great achievements.

Comments from students

- I was surprised at how quickly Mercury revolved.
- Mars was smaller than I expected.
- I can't believe that Kepler discovered the third law without using the law of universal gravitation.

I hope you'll encourage your students to experience Kepler's great achievements. These are some comments from my students. I was surprised at how quickly Mercury revolved. Mars was smaller than I expected. I can't believe that Kepler discovered the third law without using the law of universal gravitation.



Thank you for your cooperation. Did you enjoy the experiment?